3-3 Videos Guide

3-3a

- The Integral Test:
  Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$. Then the series $\sum_{n=1}^{\infty} a_n$ is convergent $\Leftrightarrow \int_1^{\infty} f(x)\, dx$ is convergent.

- Convergence of a p-series
  - $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$

3-3b

Exercise:

- Determine whether the series is convergent or divergent.

$$1 - 5 + \frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \frac{1}{13} + \cdots$$

3-3c

Estimating sums:

- The remainder of a partial sum and estimating sums: Suppose $f(k) = a_k$, where f is a continuous, positive, decreasing function for $x \geq n$ and $\sum a_n$ is convergent with sum s. Then if S_n is a partial sum,
  - $R_n = s - s_n = a_{n+1} + a_{n+2} + \cdots$ is the remainder in approximating s
  - $\int_{n+1}^{\infty} f(x)\, dx \leq R_n \leq \int_n^{\infty} f(x)\, dx$
  - $s_n + \int_{n+1}^{\infty} f(x)\, dx \leq s \leq s_n + \int_n^{\infty} f(x)\, dx$

3-3d

Exercise:

a) Find the partial sum s_{10} of the series $\sum_{n=1}^{\infty} 1/n^4$. Estimate the error in using s_{10} as an approximation to the sum of the series.

b) Use $s_n + \int_{n+1}^{\infty} f(x)\, dx \leq s \leq s_n + \int_n^{\infty} f(x)\, dx$ with $n = 10$ to give an improved estimate of the sum.

c) Compare your estimate in part (b) with the exact value $\zeta(4) = \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$. (This is known as the Riemann zeta function and is used in physics and higher-level math.)

d) Find a value of n that will ensure that the error in the approximation $s \approx s_n$ is less than 0.001.

3-3e

Proof:

- The Integral Test